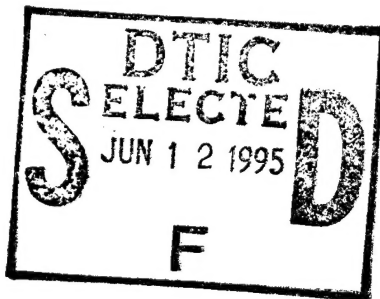


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TECHNICAL REPORT ARCCB-TR-95011

RIFLING TWIST DESIGN



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FEBRUARY 1995



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The author would like to thank David Finlayson of Benét Laboratories for suggesting that this rifling analysis focus on projectile velocity, since (for a given round) it is roughly invariant with respect to any form of rifling.

INTRODUCTION

There are many aspects to the design of rifling in guns. Here, we are concerned with only one--specification of the rifling angle or slope to produce a desired projectile torque. A rigid gun and a pre-engraved projectile are assumed. No frictional, thermal, or plastic effects are considered.

Constant twist rifling has been in use ever since the earliest rifled cannon, and in early cannon which used pre-engraved projectiles, constant twist rifling was mandatory. With the advent of the soft metal rotating band, however, it became possible to vary the rifling angle for various reasons.

Progressive twist rifling, for instance, can be used to reduce projectile torque near the origin of rifling where the base pressure is highest, while constant twist rifling produces a torque which is proportional to the base pressure. Progressive twist can be particularly important after wear has taken place near the origin of rifling. Progressive twist can still engrave the rotating band reliably, while the velocity gained by the projectile over the worn section of a constant twist gun can ruin the rotating band as it hits the unworn area.

Unfortunately, most efforts at variable twist rifling design have used analytical forms for the rifling curve and then computed the resulting torque in retrospect. This trial and error method may have been convenient before the advent of computers, but it is now an unnecessarily restrictive design technique. For instance, although traditional progressive twist rifling does decrease torque near the origin of rifling, it raises the torque considerably near the muzzle, whereas constant twist rifling has a considerably lower torque near the muzzle due to the lower base pressure there.

It would seem that we cannot get everything that we want, namely lower torques near the origin of rifling and no torque at all as the projectile exits the muzzle. Fortunately, what seems to be the case is not the case at all. In fact, if we are prepared to spend a few seconds of computer time doing some simple numerical integrations, we can have virtually any shape torque curve we want. Another comment is perhaps in order for the uninitiated: although the purpose of rifling and the torque produced thereby is to spin stabilize the projectile in flight for far greater accuracy, the shape of the torque curve has absolutely nothing whatsoever to do with the amount of spin imparted to the projectile in flight. The only determinant of this spin is the slope of the rifling curve at the muzzle. As long as this slope at this single point is held constant, the exit spin will remain constant and we are free to pick virtually any shape torque curve we want.

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RIFLING SYNTHESIS

Let $P(x)$ be the pressure travel curve, A be the bore area, r be the bore radius, and m be the projectile mass. The movement of the projectile through the rifled bore is described by the following equations:

$$PA - n \sin \phi = m \frac{dv}{dt}$$

$$L = nr \cos \phi = i \frac{d^2 \theta}{dt^2}$$

where n is the collective normal force on the lands, ϕ is the rifling angle, θ is the amount of rotation of the projectile, and i is the projectile moment of inertia. L is the torque applied to the projectile and v is its velocity.

Let $f(x)$ be the rifling curve which is related to the amount of rotation by

$$f(x) = r \theta(x)$$

First,

$$\frac{d\theta}{dt} = v \frac{d\theta}{dx} = \frac{v}{r} f'(x)$$

$$\frac{d^2 \theta}{dt^2} = \frac{v}{r} \frac{d}{dx} (v f'(x))$$

and

$$\frac{dv}{dt} = v \frac{dv}{dx} = \frac{1}{2} \frac{d}{dx} v^2$$

Hence, the equations of motion become

$$AP(x) - n \sin \phi = \frac{1}{2} m \frac{d}{dx} v^2$$

$$L = nr \cos \phi = \frac{iv}{r} \frac{d}{dx} (v f'(x))$$

Solving for n in the second equation gives us

$$n = \frac{iv}{r^2 \cos \phi} \frac{d}{dx} (v f'(x))$$

Inserting this in the first equation yields

$$AP(x) - \frac{iv}{r^2} \tan \phi \frac{d}{dx} (vf'(x)) = \frac{1}{2} m \frac{d}{dx} v^2$$

but

$$\tan \phi = f'(x)$$

Hence, we have

$$AP(x) = \frac{1}{2} m \frac{d}{dx} v^2 + \frac{i}{2r^2} \frac{d}{dx} (vf'(x))^2$$

or

$$\frac{2A}{m} P(x) = \frac{d}{dx} \left(v^2 + \frac{i}{mr^2} (vf'(x))^2 \right)$$

Integrating this, we have

$$\frac{2A}{m} \int_0^x P(u) du = \left(1 + \left(\frac{R}{r} f'(x) \right)^2 \right) v(x)^2$$

where R is the projectile radius of gyration. Hence, projectile velocity is given by

$$v(x) = \sqrt{\frac{\frac{2A}{m} \int_0^x P(u) du}{1 + \left(\frac{R}{r} f'(x) \right)^2}}$$

Let l be the length of travel (of the rotating band). $P(x)$ is the projectile base pressure when the rotating band is at x . A good typical value for

$$\frac{R}{r} f'(l)$$

is 0.1. We therefore see that the muzzle velocity of a rifled gun is about one-half of one percent less than the smoothbore muzzle velocity. Indeed we can see as Finlayson points out that the projectile velocity is roughly invariant with respect to virtually all forms of rifling (ref 1).

Now, define the torque function to be the product of some constant and some torque shape function:

$$L(x) = c \lambda(x)$$

Recalling that

$$\frac{d}{dx}(v(x)f'(x)) = \frac{rL(x)}{iv(x)}$$

We have

$$\frac{d}{dx}(v(x)f'(x)) = \frac{cr}{i} \frac{\lambda(x)}{v(x)}$$

and integrating this equation, we get

$$f'(x) = \frac{cr}{i} \frac{1}{v(x)} \int_0^x \frac{\lambda(u)}{v(u)} du$$

Now let

$$I(x) = \sqrt{\int_0^x P(u) du}$$

and

$$U(x) = \sqrt{1 + \left(\frac{R}{r} f'(x)\right)^2}$$

Hence, v is given by

$$v(x) = \sqrt{\frac{2A}{m} \frac{I(x)}{U(x)}}$$

and therefore

$$f'(x) = \frac{cmr}{2Ai} \frac{U(x)}{I(x)} \int_0^x \frac{\lambda(u)U(u)}{I(u)} du$$

Letting

$$J(x) = \int_0^x \frac{\lambda(u)U(u)}{I(u)} du$$

We have

$$f'(x) = \frac{cr}{2AR^2} \frac{U(x)J(x)}{I(x)}$$

And hence, when $f'(l)$ has been specified, c is defined as

$$c = \frac{2AR^2 f'(l)I(l)}{rU(l)J(l)}$$

Therefore

$$f'(x) = \frac{f'(l)I(l)U(x)J(x)}{U(l)J(l)I(x)}$$

Rewriting this last equation in the form

$$f'(x) = f'(l) \frac{U(x)}{U(l)} g(x)$$

where

$$g(x) = \frac{I(l)J(x)}{J(l)I(x)}$$

we have

$$f'(x)^2 = f'(l)^2 \frac{1 + \left(\frac{R}{r} f'(x)\right)^2}{1 + \left(\frac{R}{r} f'(l)\right)^2} g(x)^2$$

which we can solve for $f'(x)$, getting

$$f'(x) = \frac{f'(l)g(x)}{\sqrt{1 + \left(\frac{R}{r} f'(l)\right)^2 (1 - g(x)^2)}}$$

Now, for the sake of numerical accuracy in computing the integral $J(x)$ (especially for small values of x), it is necessary to perform an integration by parts on J . Recall that J is given by

$$J(x) = \int_0^x \frac{\lambda(u)U(u)}{I(u)} du$$

We must have

$$\lambda(0) = 0$$

for this integral to exist, since I is proportional to x for small values of x . Since

$$I(x)^2 = \int_0^x P(u) du$$

we have

$$I(x) = \frac{P(x)}{2I'(x)}$$

Hence,

$$J(x) = \int_0^x \frac{2\lambda(u)U(u)}{P(u)} I'(u) du = \int_0^x K(u) dI(u)$$

where

$$K(x) = \frac{2\lambda(x)U(x)}{P(x)}$$

Note that

$$K(0) = \frac{2\lambda'(0)U(0)}{P'(0)}$$

by l'Hospital's rule.

Using integration by parts on J , we have

$$J(x) = K(x)I(x) - K(0)I(0) - S(x) = K(x)I(x) - S(x)$$

where

$$S(x) = \int_0^x I(u) dK(u)$$

The Stieltjes integral S should be computed directly by trapezoidal rule because K may not be differentiable. Indeed, K need not even be continuous.

Now, in order to compute $g(0)$, it is sufficient to note the asymptotic behavior of λ , P , I , and J for small values of x :

$$\lambda(x) \sim x \lambda'(0)$$

$$P(x) \sim x P'(0)$$

$$I(x)^2 = \int_0^x P(u) du \sim \frac{1}{2} x^2 P'(0)$$

$$J(x) \sim x \lambda'(0) U(0) \sqrt{\frac{2}{P'(0)}}$$

We therefore have

$$\frac{J(x)}{I(x)} \sim \frac{2 \lambda'(0) U(0)}{P'(0)} = K(0)$$

and

$$g(0) = \frac{K(0) I(l)}{J(l)}$$

We now summarize in one place the equations applicable to rifling synthesis.

$$P(0) = 0 \quad \lambda(0) = 0$$

$$I(x) = \sqrt{\int_0^x P(u) du}$$

$$U(x) = \sqrt{1 + \left(\frac{R}{r} f'(x) \right)^2}$$

$$K(x) = \frac{2 \lambda(x) U(x)}{P(x)}$$

$$K(0) = \frac{2 \lambda'(0) U(0)}{P'(0)}$$

$$S(x) = \int_0^x I(u) dK(u)$$

$$J(x) = K(x) I(x) - S(x)$$

$$g(x) = \frac{I(l)J(x)}{J(l)I(x)}$$

$$g(0) = \frac{K(0)I(l)}{J(l)}$$

$$f'(x) = f'(l) \frac{U(x)}{U(l)} g(x)$$

$$f'(x) = \frac{f'(l)g(x)}{\sqrt{1 + \left(\frac{R}{r}f'(l)\right)^2 (1 - g(x)^2)}}$$

$$f(x) = \int_0^x f'(u) du$$

The equations defining $f'(x)$ do so implicitly because $f'(x)$ enters the right-hand sides through U . Functional iteration is used to obtain $f'(x)$. Convergence is extremely rapid, taking only a few iterations. In fact, if we take $f'(x) \equiv f'(l)$ initially, the very first iteration gives a result which is in error by only about one quarter of a percent for most reasonable λ 's. The reason for this is that U is only very weakly dependent on $f'(x)$, since the square of the latter is small relative to unity. Hence, U is roughly a constant near unity for all practical rifling configurations.

Also note from the equations that for the first iteration, U is a constant which cancels out. Hence the rifling slope function is determinable within about a quarter of a percent error without knowledge of either the projectile radius or radius of gyration. The absolute values of pressure are also unnecessary. Only the shapes of the pressure travel function and the desired torque function are necessary to determine a good approximation to the rifling configuration for any size gun with any muzzle twist and any projectile regardless of its mass or moment of inertia!

FIGURES OF MERIT

Let us take

$$\lambda(x) = kP(x)$$

For the first (and last) iteration, set

$$f'(x) \equiv f'(l)$$

Hence,

$$U(x) = \text{const} = U$$

$$K(x) = 2kU = \text{const}$$

$$S(x) = \int_0^x I dK = 0$$

$$J(x) = 2kUI(x)$$

$$g(x) = \frac{I(l)2kUI(x)}{2kUI(l)I(x)} = 1$$

Therefore

$$f'(x) \equiv f'(l)$$

(immediate convergence to constant twist rifling).

Now, recall that

$$\text{Torque} = L = c \lambda$$

where

$$c = \frac{2AR^2 f'(l)I(l)}{rU(l)J(l)}$$

Denote the max norm by $\| \cdot \|$. The maximum torque force will then be

$$\frac{\|L\|}{r} = \frac{c \|\lambda\|}{r}$$

and the maximum pressure force is $\|P\|A$.

We define μ to be the ratio of maximum torque force to maximum pressure force:

$$\mu = \frac{c \|\lambda\|}{\|P\|Ar} = \frac{2 \|\lambda\| \left(\frac{R}{r}\right)^2 f'(l)I(l)}{\|P\|U(l)J(l)}$$

For the constant twist case ($\lambda \propto P$), we have

$$\mu_c = \frac{2kJP \left(\frac{R}{r}\right)^2 f'(l) I(l)}{JP U(l) 2k U(l) I(l)} = \frac{\left(\frac{R}{r}\right)^2 f'(l)}{U(l)^2}$$

We can also define another figure of merit (σ) as the ratio of maximum torque to maximum constant twist torque:

$$\sigma = \frac{\mu}{\mu_c} = \frac{2JP I(l) U(l)}{JP J(l)}$$

Note that $\sigma=1$ in the constant twist case and that σ is virtually independent of R , r , and $f'(l)$. We can now compare different torque shape functions by comparing their μ 's and σ 's.

We would presumably like these figures of merit to be acceptably small (up to a point). Now, the smallest values for μ and σ one can typically get (depending on the pressure travel curve shape) are about 0.03 and 0.45, respectively. We get these rather small values by letting λ be constant except in a small neighborhood of $x=0$ where λ' must be quite large. Unfortunately, the byproduct of this λ is an f' having $f'(0)$ considerably larger than $f'(l)$, an obviously impractical situation. However, if we let λ be proportional to P up to maximum pressure and constant everywhere else except for the last five percent of travel where we drop λ linearly down to zero, we get a μ of about 0.04 and a σ of about 0.6 with an $f'(0)$ of about $0.6 f'(l)$. The rifling is therefore progressive, but not radically so, and it substantially reduces maximum torque on the projectile while giving it a torqueless exit.

PRESSURE TRAVEL FUNCTIONS

Ideally, one might want to work with the best available pressure travel data for a given round; however, the need will generally exist to fire more than one type of round or charge in the gun. If each round type displayed the same shape pressure travel function, the rifling configuration could be determined uniquely. In general, however, the pressure travel function will vary in shape for rounds of different types and it will be necessary to use some representative or average pressure travel shape function. This being the case, it makes sense to approximate the pressure travel data with some smooth closed form function.

If the reader has some favorite numerical method for approximating pressure travel data, this section can be completely ignored. In any case, no claim of superiority is made for the simple approximation offered here; it at least suffices for testing the software associated with the problem at hand.

A simple approximation one might use is

$$P(x) = axe^{-\left(\frac{x}{b}\right)^c}$$

To determine the three parameters a , b , and c , one might enforce the three conditions

$$P(x_1) = p_1$$

$$P'(x_1) = 0$$

$$P(x_2) = p_2$$

This would allow us to put the peak pressure (p_1) where we wanted it (x_1) and specify the pressure (p_2) at the muzzle (x_2).

This representation seems insufficiently flexible, however, so the following piecewise exponential representation is suggested:

$$P(x) = \begin{cases} P_1(x) & \text{if } 0 \leq x \leq x_2 \\ P_2(x) & \text{if } x_2 \leq x \leq x_3 \end{cases}$$

where

$$P_1(x) = a_1 x e^{-\left(\frac{x}{b_1}\right)^{c_1}}$$

$$P_2(x) = a_2 + b_2 e^{-\left(\frac{x-x_2}{c_2}\right)}$$

and the six parameters a_1, b_1, c_1, a_2, b_2 , and c_2 are determined by the conditions

$$P_1(x_1) = p_1$$

$$P_1'(x_1) = 0$$

$$P_1(x_2) = p_2$$

$$P_2(x_2) = p_2$$

$$P_1'(x_2) = P_2'(x_2)$$

$$P_2(x_3) = p_3$$

Determining the parameters in $P_1(x)$:

$$P_1(x) = a_1 x e^{-\left(\frac{x}{b_1}\right)^{c_1}}$$

$$P_1'(x) = a_1 \left(1 - c_1 \left(\frac{x}{b_1}\right)^{c_1}\right) e^{-\left(\frac{x}{b_1}\right)^{c_1}}$$

$$P_1'(x_1) = 0$$

→

$$b_1 = x_1 c_1^{\frac{1}{c_1}}, \quad \left(\frac{x}{b_1}\right)^{c_1} = \frac{1}{c_1} \left(\frac{x}{x_1}\right)^{c_1}$$

and

$$P_1(x_1) = p_1 \rightarrow a_1 = \frac{p_1 e^{\frac{1}{c_1}}}{x_1}$$

We therefore have

$$P_1(x) = \frac{p_1 x}{x_1} e^{\frac{1}{c_1} \left(1 - \left(\frac{x}{x_1}\right)^{c_1}\right)}$$

in which we must now determine c_1 .

We have the order relationships

$$x_1 < x_2 < x_3, \quad p_1 > p_2 > p_3$$

Therefore

$$P_1(x_2) = p_2 \rightarrow$$

$$1 > \frac{x_1 p_2}{x_2 p_1} = e^{\frac{1}{c_1} \left(1 - \left(\frac{x_2}{x_1}\right)^{c_1}\right)}$$

or

$$\ln\left(\frac{x_1 p_2}{x_2 p_1}\right) = \frac{1}{c_1} \left(1 - \left(\frac{x_2}{x_1}\right)^{c_1}\right)$$

Letting

$$\alpha = \frac{x_2}{x_1} > 1, \quad \beta = \ln\left(\frac{x_1 p_2}{x_2 p_1}\right) < 0$$

We have the equation which must be solved for c_1 :

$$\alpha^{c_1} + \beta c_1 - 1 = 0$$

Define function f as

$$f(x) = \alpha^x + \beta x - 1$$

Clearly,

$$f(c_1) = 0, \quad f(0) = 0$$

and since $\alpha > 1$, $f(\infty) = \infty$.

Differentiating, we have

$$f'(x) = \alpha^x \ln \alpha + \beta$$

$$f''(x) = \alpha^x (\ln \alpha)^2 > 0$$

Hence,

$$f'(0) = \ln \alpha + \beta = \ln\left(\frac{x_2}{x_1}\right) + \ln\left(\frac{x_1 p_2}{x_2 p_1}\right) = \ln\left(\frac{p_2}{p_1}\right) < 0$$

Therefore, since $f(0)=0$, $f'(0) < 0$, $f''(x) > 0$ and $f(\infty)=\infty$, a positive zero of f is guaranteed. Also, there must be an m such that $f'(m)=0$. Hence,

$$\alpha^m \ln \alpha + \beta = 0, \quad m = -\frac{\ln\left(\frac{-\beta}{\ln \alpha}\right)}{\ln \alpha}$$

Expanding f in a Taylor series around m , we have

$$f(x) \approx f(m) + \frac{1}{2}f''(m)(x-m)^2$$

Hence,

$$f(z) = 0 \rightarrow$$

$$f(m) + \frac{1}{2}f''(m)(z-m)^2 \approx 0, \quad z \approx m + \sqrt{\frac{-2f(m)}{f''(m)}}$$

This is our first estimate of z (or c_1) which we will subsequently use in Newton iteration to obtain z exactly.

Now,

$$k = \frac{-\beta}{\ln \alpha} \rightarrow$$

$$m = \frac{\ln k}{\ln \alpha}$$

$$f(m) = \alpha^m + \beta m - 1 = e^{m \ln \alpha} + \frac{\beta \ln k}{\ln \alpha} - 1 = k - k \ln k - 1$$

$$f''(m) = \alpha^m (\ln \alpha)^2 = e^{m \ln \alpha} (\ln \alpha)^2 = k (\ln \alpha)^2$$

$$\frac{-2f(m)}{f''(m)} = \frac{2(\ln k + \frac{1}{k} - 1)}{(\ln \alpha)^2}$$

Therefore the first estimate of z reduces to

$$z \approx \frac{1}{\ln \alpha} (\ln k + \sqrt{2(\ln k + \frac{1}{k} - 1)})$$

And the Newton iteration for z is

$$z \leftarrow z - \frac{f(z)}{f'(z)} = z - \frac{\alpha^z + \beta z - 1}{\alpha^z \ln \alpha + \beta} = \frac{\alpha^z (z \ln \alpha - 1) + 1}{\alpha^z \ln \alpha + \beta}$$

Now, determining the parameters of $P_2(x)$:

$$P_2(x) = a_2 + b_2 e^{-\left(\frac{x-x_2}{c_2}\right)}$$

$$P_2'(x) = -\frac{b_2}{c_2} e^{-\left(\frac{x-x_2}{c_2}\right)}$$

So,

$$P_2'(x_2) = P_1'(x_2) = p_2' \rightarrow b_2 = -c_2 p_2'$$

and

$$P_2(x_2) = p_2 \rightarrow a_2 = p_2 - b_2 = p_2 + c_2 p_2'$$

Therefore, we have

$$P_2(x) = p_2 + c_2 p_2' \left(1 - e^{-\left(\frac{x-x_2}{c_2}\right)}\right)$$

Now,

$$\delta = x_3 - x_2 > 0, \quad P_2(x_3) = p_3 \rightarrow$$

$$c_2 p_2' \left(1 - e^{-\frac{\delta}{c_2}}\right) + p_2 - p_3 = 0$$

which must be solved for c_2 .

Define f by

$$f(x) = x p_2' \left(1 - e^{-\frac{\delta}{x}}\right) + p_2 - p_3$$

Clearly,

$$f(c_2) = 0, \quad f(0) = p_2 - p_3 > 0$$

and for large values of x ,

$$e^{-\frac{\delta}{x}} \sim 1 - \frac{\delta}{x}$$

Therefore,

$$f(\infty) = \delta p_2' + p_2 - p_3$$

Now,

$$f'(x) = p_2' \left(1 - \left(1 + \frac{\delta}{x} \right) e^{-\frac{\delta}{x}} \right)$$

and since,

$$\lim_{x \rightarrow 0} \frac{e^{-\frac{\delta}{x}}}{x} = 0$$

we have

$$f'(0) = p_2' < 0$$

Also, for large values of x ,

$$e^{-\frac{\delta}{x}} \sim 1 - \frac{\delta}{x} \Rightarrow f'(x) \sim \frac{\delta^2 p_2'}{x^2} \leq 0 \Rightarrow f'(\infty) = 0$$

Furthermore,

$$f''(x) = -\frac{\delta^2 p_2'}{x^3} e^{-\frac{\delta}{x}} \geq 0$$

Therefore, f' is monotone increasing and can never be positive. We can therefore finally conclude that f is monotone decreasing and is guaranteed to have exactly one positive zero provided

$$f(\infty) = \delta p_2' + p_2 - p_3 < 0$$

The Newton iteration for $f(z)=0$ is

$$z \leftarrow z - \frac{f(z)}{f'(z)}$$

Taking $z=0$ initially, we have

$$z \approx \frac{f(0)}{f'(0)} = \frac{p_3 - p_2}{p_2'}$$

as our first guess at z .

Computing

$$zf'(z) - f(z) = p_3 - p_2 - \delta p_2' e^{-\frac{\delta}{z}}$$

we have the Newton iteration for the correct z :

$$z \leftarrow \frac{p_3 - p_2 - \delta p_2' e^{-\frac{\delta}{z}}}{p_2' \left(1 - \left(1 + \frac{\delta}{z} \right) e^{-\frac{\delta}{z}} \right)}$$

Summarizing the results of this section:

$$P(x) = \begin{cases} P_1(x) & \text{if } 0 \leq x \leq x_2 \\ P_2(x) & \text{if } x_2 < x \leq x_3 \end{cases}$$

$$g(x) = 1 - \left(\frac{x}{x_1} \right)^{c_1}$$

$$P_1(x) = \frac{p_1 x}{x_1} e^{\frac{g(x)}{c_1}}$$

$$P_1'(x) = \frac{p_1}{x_1} g(x) e^{\frac{g(x)}{c_1}}$$

$$\alpha = \frac{x_2}{x_1}$$

$$\beta = \ln \left(\frac{x_1 p_2}{x_2 p_1} \right)$$

$$k = \frac{-\beta}{\ln \alpha}$$

$$c_1 \approx \frac{1}{\ln \alpha} (\ln k + \sqrt{2(\ln k + \frac{1}{k} - 1)})$$

$$c_1 = \frac{\alpha^{c_1} (c_1 \ln \alpha - 1) + 1}{\alpha^{c_1} \ln \alpha + \beta}$$

$$P_2(x) = p_2 + c_2 p_2' \left(1 - e^{-\frac{x-x_2}{c_2}} \right)$$

$$p_2' = P_1'(x_2)$$

$$\delta = x_3 - x_2$$

$$\text{need } p_2' < \frac{p_3 - p_2}{\delta}$$

$$c_2 \approx \frac{p_3 - p_2}{p_2'}$$

$$c_2 = \frac{p_3 - p_2 - \delta p_2' e^{-\frac{\delta}{c_2}}}{p_2' \left(1 - \left(1 + \frac{\delta}{c_2} \right) e^{-\frac{\delta}{c_2}} \right)}$$

RIFLING ANALYSIS

For the sake of completeness, we briefly derive the equations to compute torque and pressure in terms of rifling twist. Recalling that

$$\begin{aligned} L &= \frac{iv}{r} \frac{d}{dx} (vf'(x)) = \frac{i}{2rf'(x)} 2vf'(x) \frac{d}{dx} (vf'(x)) \\ &= \frac{i}{2rf'(x)} \frac{d}{dx} (vf'(x))^2 \end{aligned}$$

and further recalling that

$$v(x) = \sqrt{\frac{2A}{m} \frac{I(x)}{U(x)}}$$

we have

$$\begin{aligned} L(x) &= \frac{i}{2rf'(x)} \frac{d}{dx} \left(\frac{2A}{m} \frac{f'(x)^2 I(x)^2}{U(x)^2} \right) \\ &= \frac{AR^2}{rf'(x)} \frac{d}{dx} \frac{f'(x)^2 \int_0^x P(u) du}{1 + \left(\frac{Rf'(x)}{r} \right)^2} \end{aligned}$$

For constant twist rifling, where $f'(x) = \text{constant} = k$, we have

$$L(x) = \frac{\pi k r R^2}{1 + \left(\frac{kR}{r} \right)^2} P(x)$$

If one wished to write P in terms of L , one could begin with

$$P = \frac{m}{2A} \frac{d}{dx} (U^2 v^2)$$

and since

$$\frac{d}{dx} (vf'(x)) = \frac{rL}{iv}$$

we have

$$\begin{aligned} 2vf'(x)\frac{d}{dx}(vf'(x)) &= \frac{2rL(x)f'(x)}{i} \\ &= \frac{d}{dx}(vf'(x))^2 \end{aligned}$$

which gives us

$$v^2f'(x)^2 = \frac{2r}{i} \int_0^x L(u)f'(u)du$$

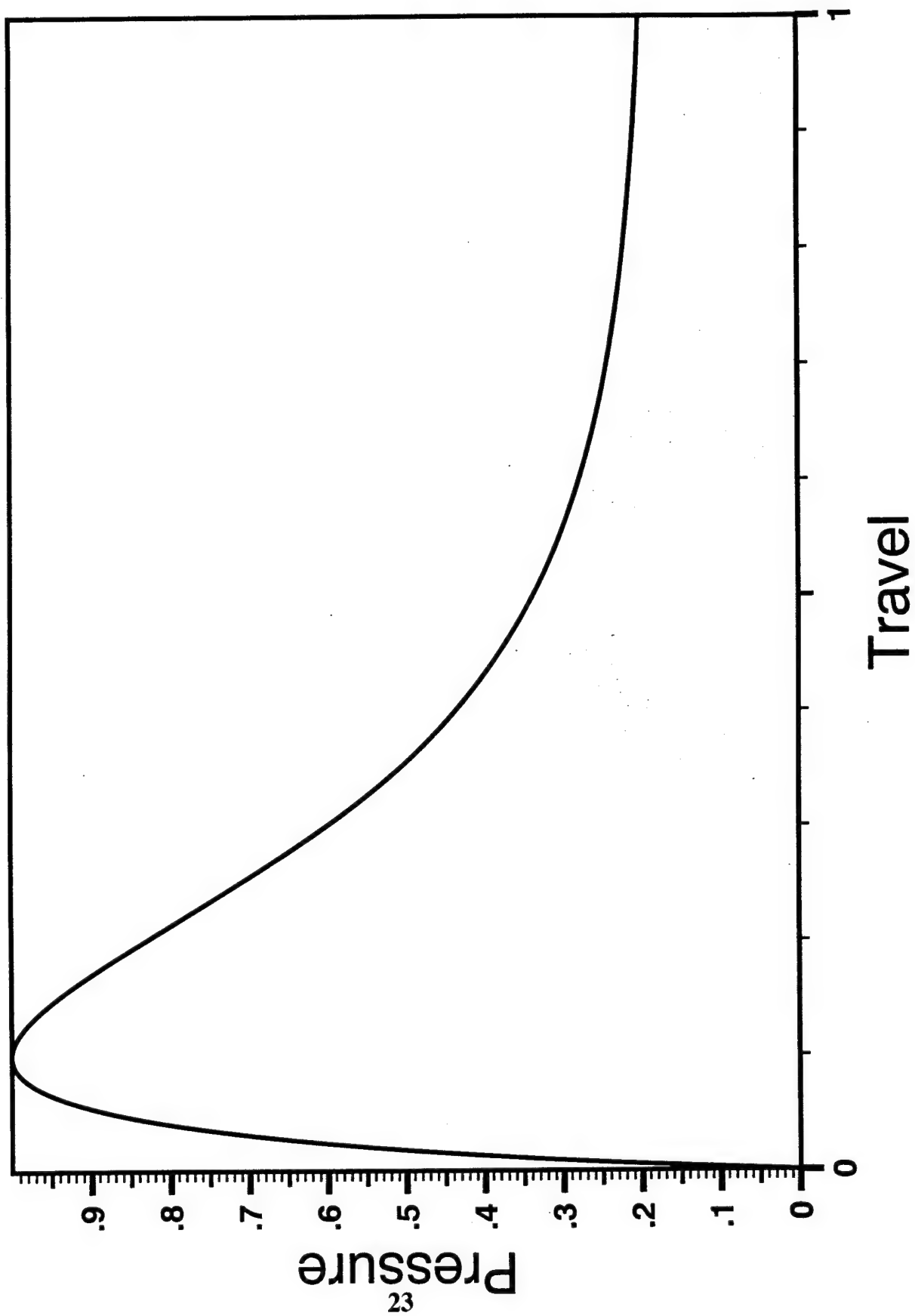
and hence that

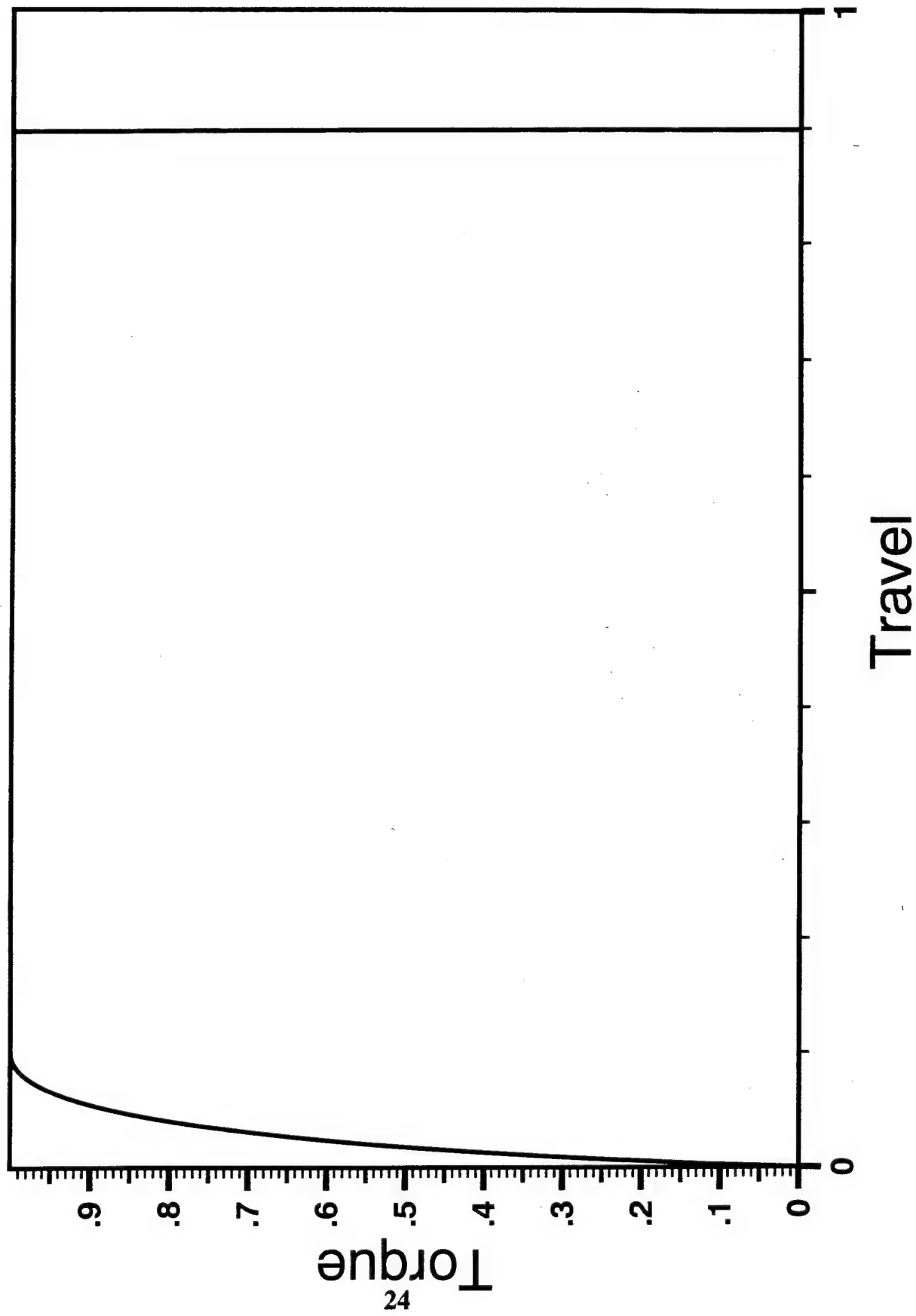
$$\begin{aligned} P(x) &= \frac{m}{2A} \frac{d}{dx} \left[\frac{2r}{i} \frac{U(x)^2}{f'(x)^2} \int_0^x L(u)f'(u)du \right] \\ &= \frac{r}{AR^2} \frac{d}{dx} \left[\left(\frac{1}{f'(x)^2} + \frac{R^2}{r^2} \right) \int_0^x L(u)f'(u)du \right] \end{aligned}$$

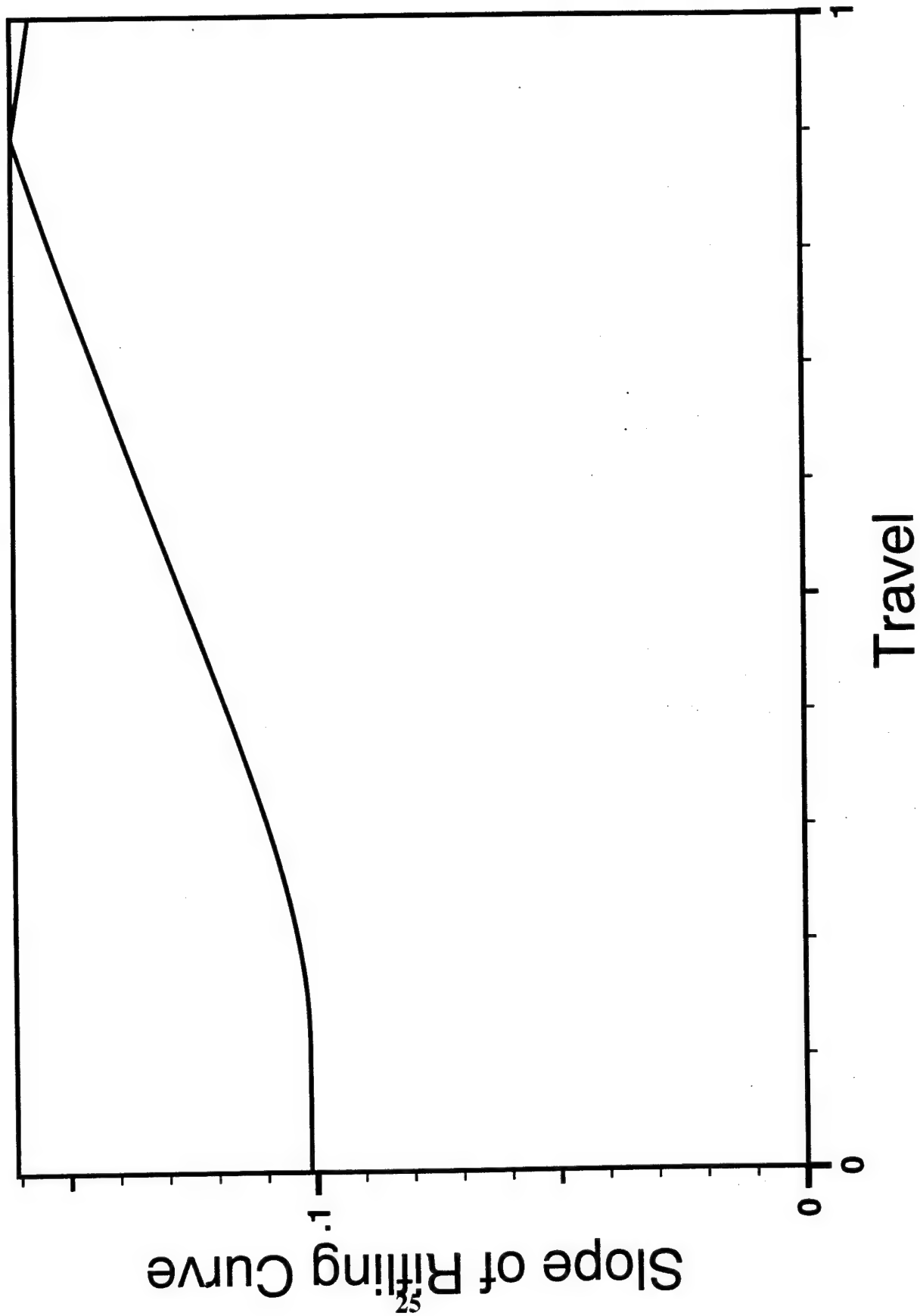
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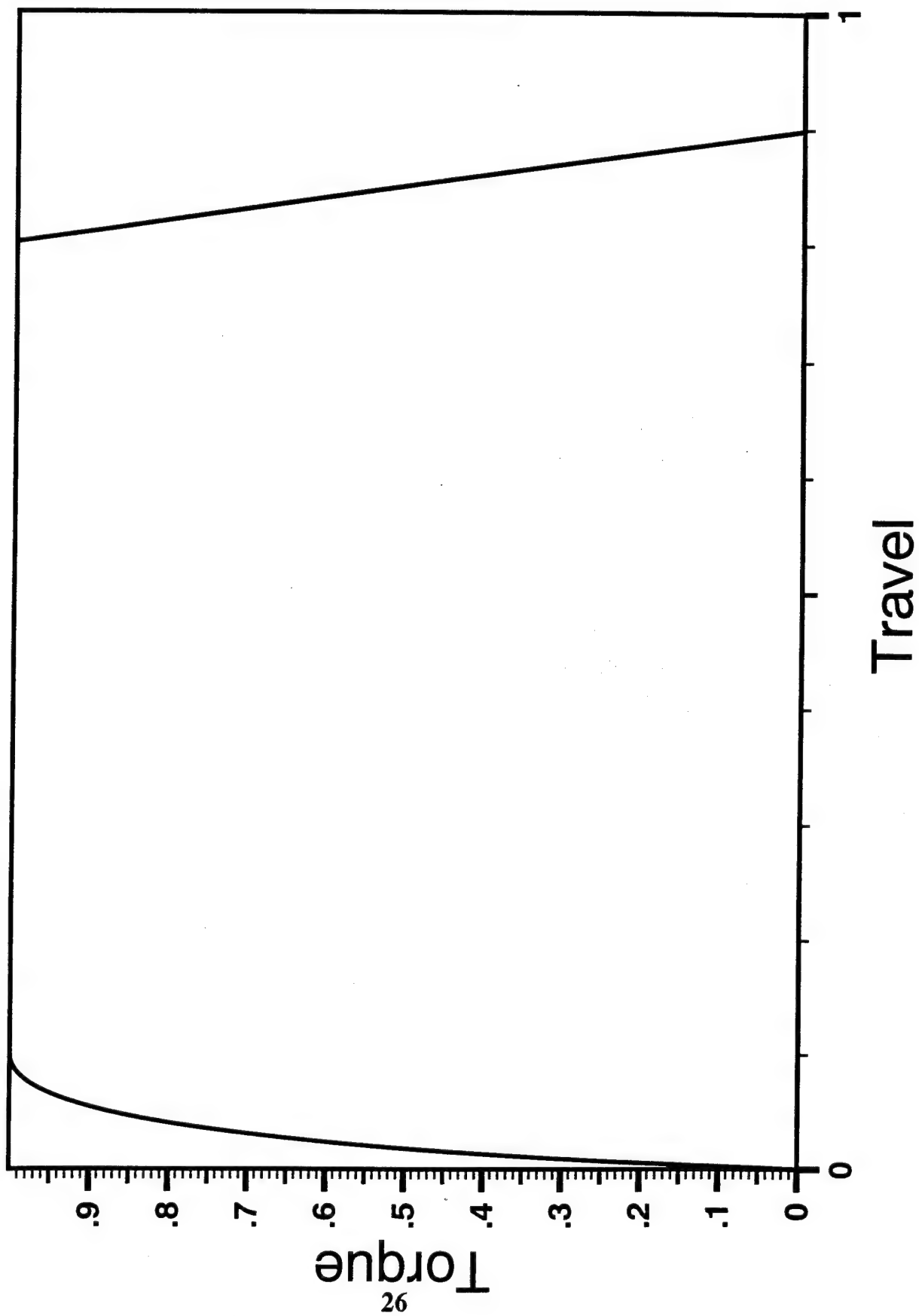
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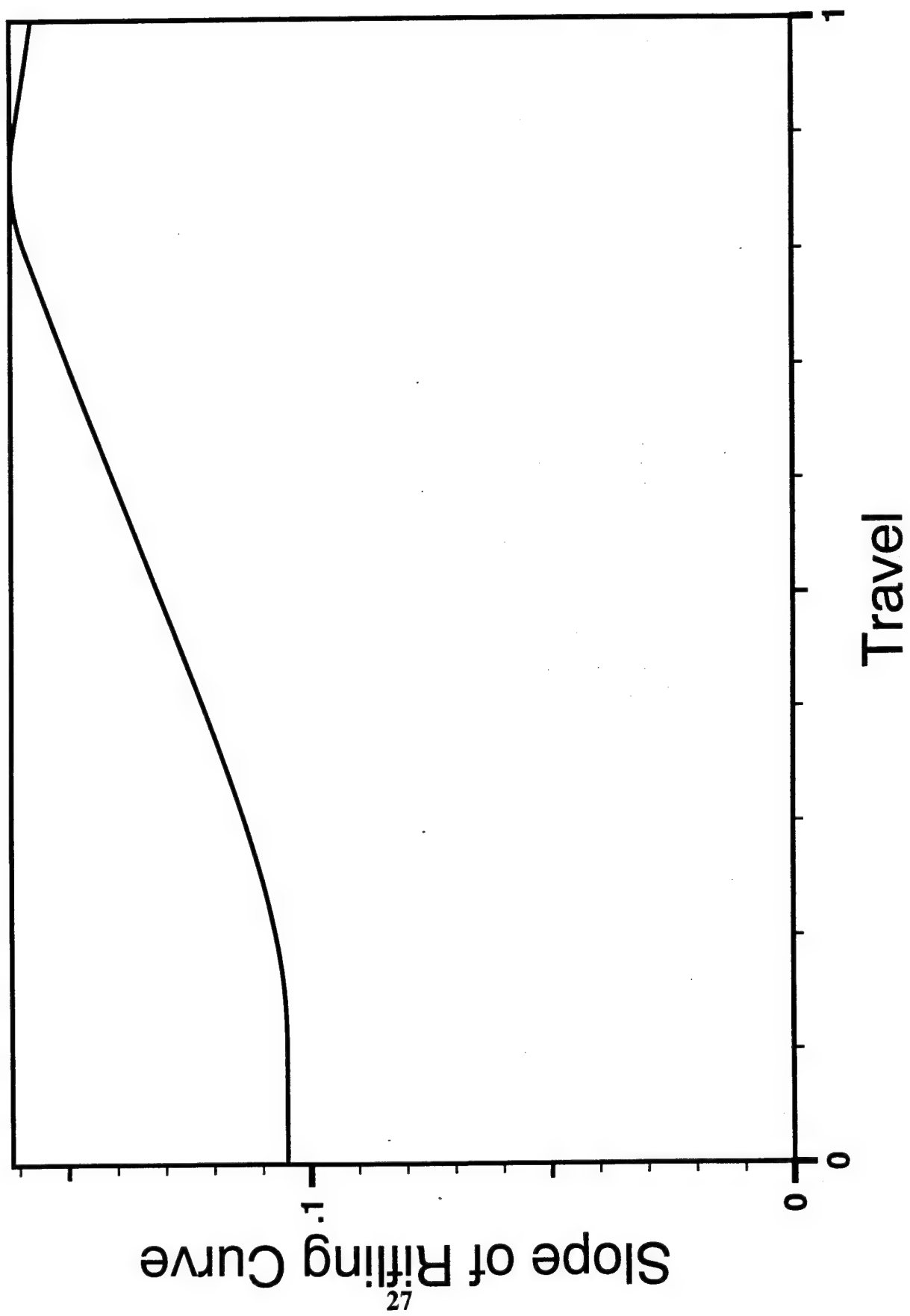
APPENDIX

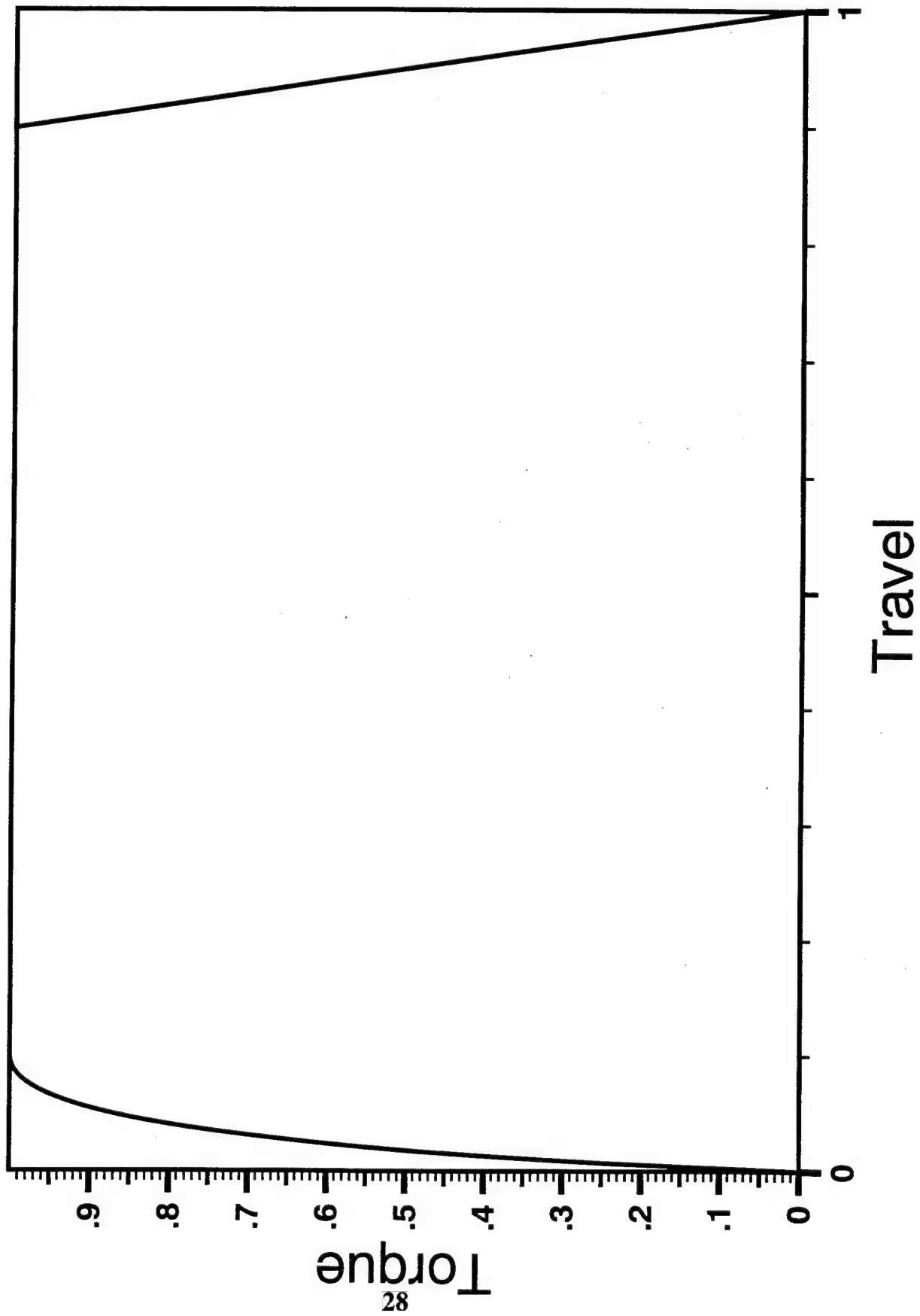


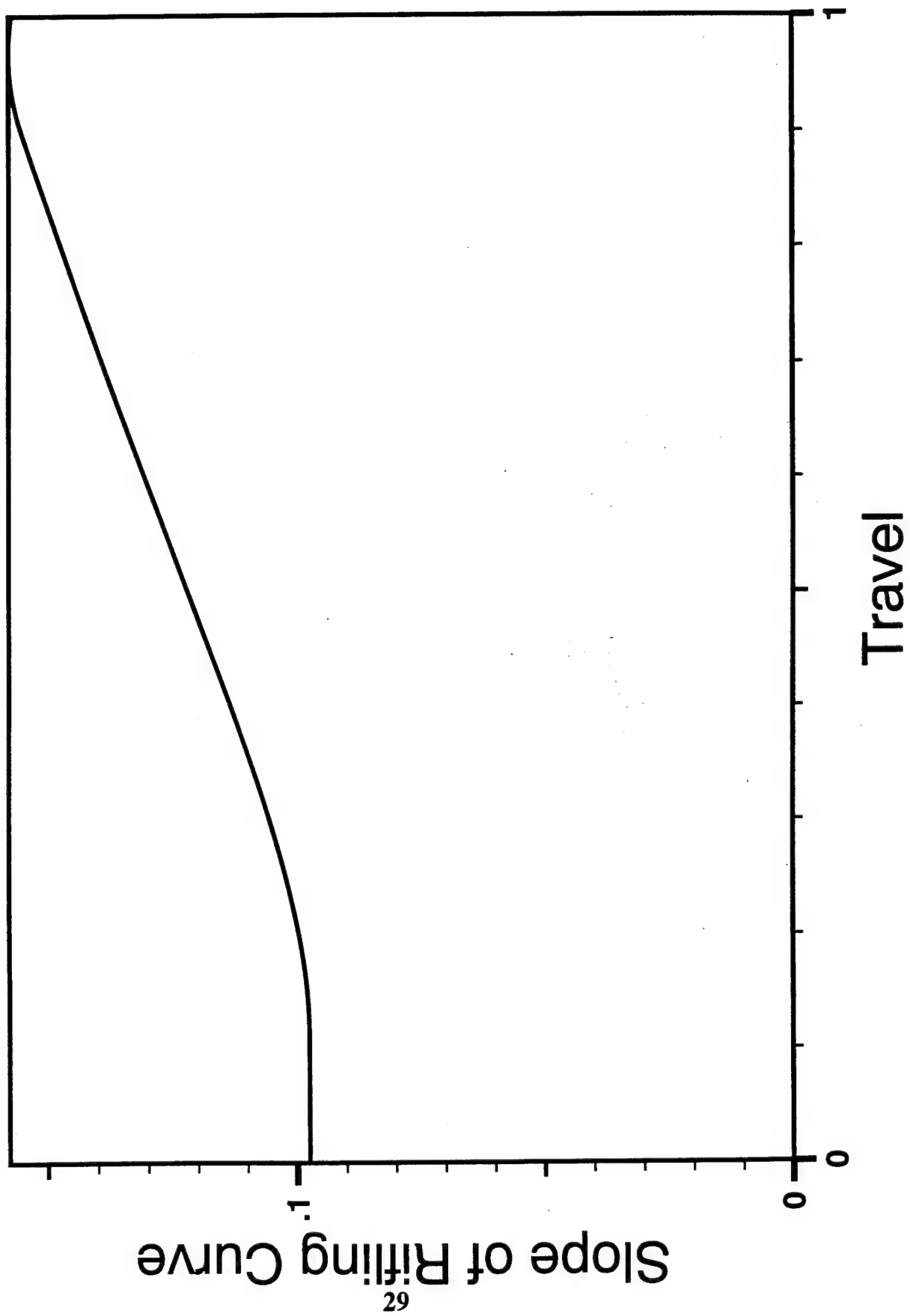


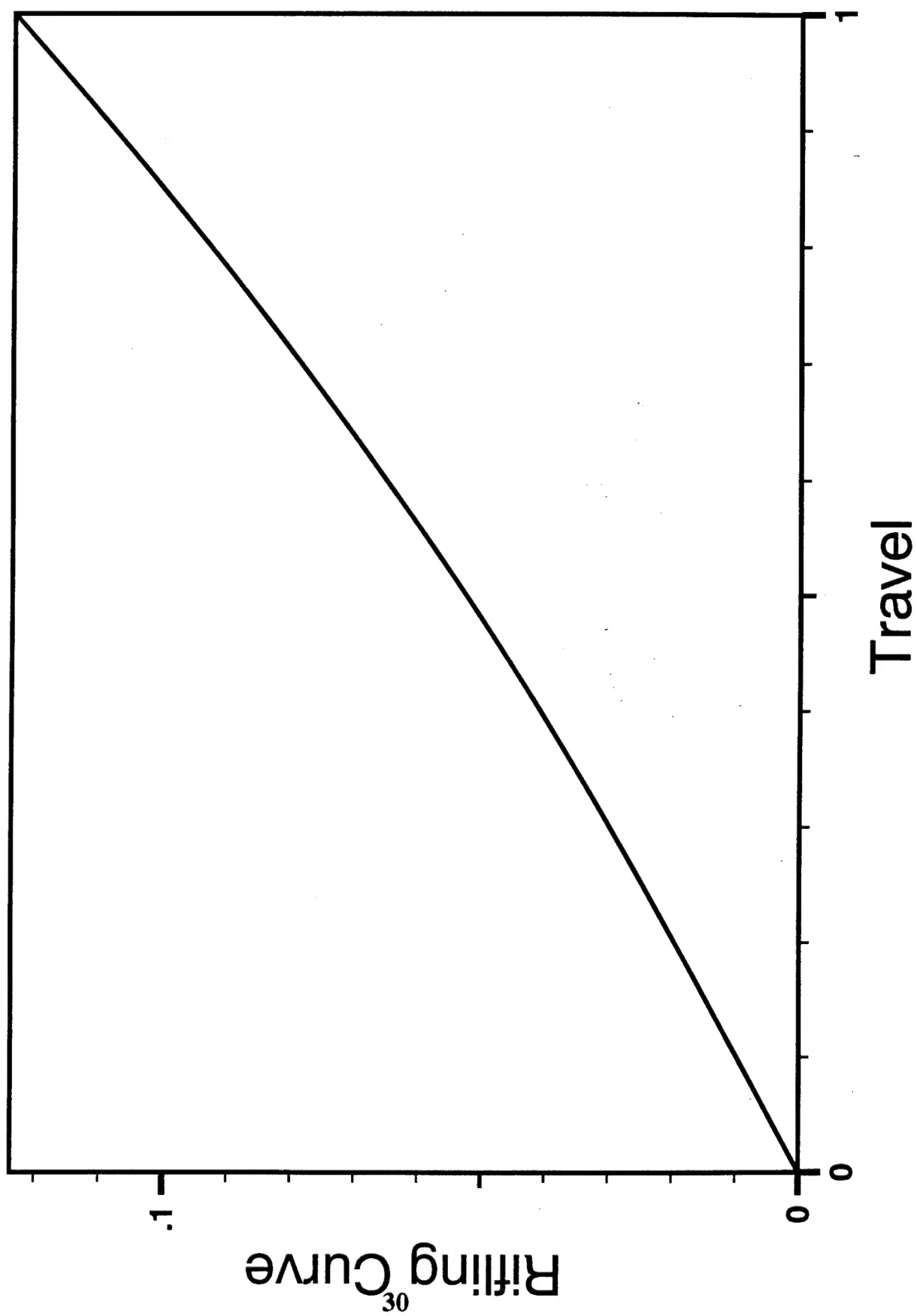












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